$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}+\frac{z_1^2}{c^2}=1=\frac{x_2^2}{a^2}+\ldots\ldots=\frac{x_3^2}{a^2}+\ \ldots\ldots (7),$$

$$\text{and } x_1y_1+x_2y_2+x_3y_3=y_1z_1+\text{etc.},=z_1x_1+\text{etc.},=0\ldots\ldots (8),$$

$$l/p=\frac{x_1+y_1+z_1}{a^2},\ m/p=\text{etc.},\ n/p=\text{etc.},\ \ldots\ldots (9).$$

These must be put into (1).

Also solved by *G. B. M. ZERR*, *J. W. YOUNG*, *LON C. WALKER*, *J. SCHEFFER*, and *GEORGE LILLEY*.

128. Proposed by W. H. CARTER, Vice President and Professor of Mathematics, Centenary College, Jackson, La.

Given $F=\triangle^{n-1}\div(n-1)!.\triangle_1.\triangle_2\ldots\ldots\triangle_n$, where $\triangle$=the determinant $(a_1b_2c_3\ldots\ldots k_n)$ and $\triangle_1\triangle_2\ldots\ldots\triangle_n$ are the minors of the elements of the nth column; and a_m, b_m, $c_m\ldots\ldots$etc., $(m=1, 2, 3\ldots\ldots n)$ are the coefficients of n given equations containing $n-1$ variables. Show (1) that $n=3$, F=the area of a triangle, and (2) if $n=4$, F=the volume of the tetrahedron.

Solution by J. W. YOUNG, Student in Ohio State University, Columbus, O.

1. Let $n=3$. The points of intersection of the three lines represented by the given equations, are

$$x_1=-\frac{A_1}{C_1};\ x_2=-\frac{A_2}{C_2};\ x_3=-\frac{A_3}{C_3};$$

$$y_1=-\frac{B_1}{C_1};\ y_2=-\frac{B_2}{C_2};\ y_3=-\frac{B_3}{C_3};$$

where, by the usual notation, A_k equals the co-factor a_k, in the determinant $\triangle$.

The area of the triangle formed by these points is

$$\text{Area}=\tfrac{1}{2}\begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}=\tfrac{1}{2}\begin{vmatrix} -\dfrac{A_1}{C_1}, & -\dfrac{B_1}{C_1}, & 1 \\ -\dfrac{A_2}{C_2}, & -\dfrac{B_2}{C_2} & 1 \\ -\dfrac{A_3}{C_3}, & -\dfrac{B_3}{C_3} & 1 \end{vmatrix}=\tfrac{1}{2}\begin{vmatrix} A_1, & B_1, & C_1 \\ A_2, & B_2, & C_2 \\ A_3, & B_3, & C_3 \end{vmatrix}\div C_1C_2C_3$$

and this, by a well-known theorem in determinants,

$$=\tfrac{1}{2}\triangle^2\div C_1C_2C_3=F.$$

2. Let $n=4$. The intersections of the four planes given by the equations are found precisely as above.

The volume of the tetrahedron found by the points is

$$\frac{1}{3\,!}\begin{vmatrix} x_1, & y_1, & z_1, & 1 \\ x_2, & y_2, & z_2, & 1 \\ x_3, & y_3, & z_3, & 1 \\ x_4, & y_4, & z_4, & 1 \end{vmatrix}$$

or substituting the values of $x_1 y_1 z_1$, etc., we have

$$\text{Volume}=\tfrac{1}{6}\begin{vmatrix} A_1, & B_1, & C_1, & D_1 \\ A_2, & B_2, & C_2, & D_2 \\ A_3, & B_3, & C_3, & D_3 \\ A_4, & B_4, & C_4, & D_4 \end{vmatrix} \div D_1 D_2 D_3 D_4$$

$$=\tfrac{1}{6}\triangle^3 \div \triangle_1 \triangle_2 \triangle_3 \triangle_4 = F.$$

Also solved by *G. B. M. ZERR*, *WALTER H. DRANE*, and the *PROPOSER*.

Professor Carter asks: What does F represent when n is greater than 4?

CALCULUS.

97. Proposed by ARTEMAS MARTIN, A.M., Ph.D., LL.D., United States Coast and Geodetic Survey Office, Washington, D. C.

An auger hole, radius r, is bored through a prolate spheroid; the axis of the auger passing through the center, perpendicular to the major axis. Find the volume removed.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $\dfrac{x^2}{a^2}+\dfrac{y^2+z^2}{b^2}=1$, be the equation to the prolate spheroid.

$x^2+y^2=r^2$, the equation to the cylinder.

$$\therefore\ V=8/a\int_0^r\int_0^{\sqrt{(r^2-x^2)}}\sqrt{[b^2(a^2-x^2)-a^2y^2]}\,dx\,dy$$

$$=4/a\int_0^r\sqrt{\{(r^2-x^2)[a^2(b^2-r^2)+(a^2-b^2)x^2]\}}\,dx$$

$$+\frac{4b^2}{a^2}\int_0^r(a^2-x^2)\sin^{-1}\left[\frac{a}{b}\sqrt{\left(\frac{r^2-x^2}{a^2-x^2}\right)}\right]dx$$

$$=4/a\int_0^r\sqrt{\{(r^2-x^2)[a^2(b^2-r^2)+(a^2-b^2)x^2]\}}\,dx$$

$$+\frac{4b^2(a^2-r^2)}{3a}\int_0^r\frac{x^2\,dx}{\sqrt{\{(r^2-x^2)[a^2(b^2-r^2)+(a^2-b^2)x^2]\}}}$$

$$+\frac{8ab^2(a^2-r^2)}{3}\int_0^r\frac{x^2\,dx}{(a^2-x^2)\sqrt{\{(r^2-x^2)[a^2(b^2-r^2)+(a^2-b^2)x^2]\}}}$$